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Free Entry under Uncertainty

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When focusing on firm's risk-aversion in industry equilibrium, the number of firms may be either larger or smaller when comparing market equilibrium with and without price uncertainty. In this paper, we introduce risk-averse firms under cost uncertainty in a model of spatial differentiation and show that the impact of uncertainty will increase the number of firms in an industry. With increased uncertainty, the risk premium of the marginal buyer increases by more than the risk premium of the average buyer, so that the price increases by more than the risk premium. When turning to the free entry game, we find that the market generates too many firms.

Keywords: spatial differentiation, risk-averse firms, cost uncertainty.

JEL Classification: D43, D81, L12.

1 Introduction

When explaining variations in the number of firms across industries, standard arguments drawing on scale economies and entry conditions usually neglect the issue of uncertainty. Unfortunately, the prevalent assumption of risk-neutral firms is not really appropriate. Several theoretical contributions have recently considered a setting where firms behave in a risk-averse manner (see Asplund, 2002, and the references therein).

There are several reasons that may explain why firms are characterized by risk aversion. Existing fixed costs mean that firms invest before producing, so that risk aversion is driven by liquidity constraints (Drèze, 1987). Many firms have an imperfect access to capital markets and have to bear part of the risk associated with their production. Another reason deals with non-diversified owners. Although owners may be tempted to maximize expected profits, the delegation of control to managers in a hierarchical structure favors the reluctance to bear risk since the managers' income is clearly related to the firm's performance. Other arguments in the prevalent literature are linked to costly financial distress and to nonlinear tax systems. Some studies have suggested that the extent of corporate hedging activities may be interpreted as a reluctance to bear risk (Nance et al., 1993; Gézci et al., 1997). Clearly, accounting for uncertainty has strong implications for the product market competition.

The pioneering work dealing with the impact of uncertainty on firms' decisions is due to Sandmo (1971). Within a partial equilibrium framework, greater price uncertainty is expected to lower the optimal quantity produced in a perfectly competitive market[1]. Then, the degree and distribution of price uncertainty are significant factors to explain industry structure. Appelbaum and Katz (1986) were the first authors to focus on the impact of risk aversion in a model in which the number of firms is determined endogenously (see also Haruna, 1996). Once a competitive equilibrium is introduced, they show that the effect of price uncertainty on the number of firms in an industry can no longer be signed, even with additional assumptions about relative or absolute risk aversion.

Despite the ambiguous prediction of price uncertainty on the industry equilibrium, it seems tempting to expect a negative relationship between uncertainty and the number of firms in an industry[2]. Intuitively, firms characterized by more risk aversion certainly prefer not to operate in a market where price uncertainty prevails (see the discussion in Sandmo, 1971). Uncertainty may be seen as a natural barrier to entry, thereby leading to a decrease in the number of firms in the industry. However, it is well known since the influential paper of Oi (1961) that variability may also offer opportunities for increasing average profits for risk-averse firms. Average profits of a price taker are increasing in the variability of the output price and Oi's conclusion does generalize to a considerable

1 Positive profits will exist in the market under price uncertainty, in a context where firms value their losses higher than their profits. Leland (1972) also accounts for the effect of uncertainty in a monopoly setting.

2 Such a negative relationship seems rather well supported by the data. Using a cross-section of US manufacturing industries, Ghosal (1996) shows that greater uncertainty has a negative impact on the number of firms in an industry when correcting for endogeneity of the price uncertainty measure.

extent (Friberg and Martensen, 2000). Such positive effects on profits could have a beneficial influence on the entry of firms.

In this paper, drawing on the previous work of Appelbaum and Katz (1986), we further examine the consequences of uncertainty within an industry equilibrium framework. We focus on the problem of free entry and exit of firms in a setting of spatial differentiation with cost uncertainty. Specifically, we draw on the location model originally proposed by Salop (1979), who introduces differentiation using a circular city with consumers uniformly distributed on its circumference. We introduce uncertainty on the marginal costs of production, in a setting where prices are set before marginal costs are realized. Our main result is that the undetermined effect of uncertainty on the number of firms in an industry does no longer hold. In a location model with horizontally differentiated products and risk-averse firms, greater marginal cost uncertainty is expected to increase the number of firms operating in the industry.

The intuition of that result is as follows. In a location model (either linear or spatial), it is well known that the competitive price under product differentiation is defined as the sum of the marginal cost and the transportation cost, which leads to a monopoly power for the different firms in the industry (see Tirole, 1988). If one introduces cost uncertainty, the optimal price includes an additional term corresponding to the risk premium faced by the firms, so that firms charge higher prices to consumers under uncertainty. Under the assumption of risk aversion, we show that the cost of uncertainty charged by firms to consumers is higher than the cost of increased uncertainty supported by firms. The risk premium of the marginal buyer is larger than the risk premium of the average buyer with more uncertainty, so that a rise in uncertainty implies that the price increases by more than the risk premium.

By focusing on uncertainty in a location model, our paper is related to the recent literature on risk-averse firms in an oligopoly. In a context of cost uncertainty, Wambach (1999) proves that the Bertrand paradox that two firms are sufficient for perfect competition does no longer hold with risk-averse firms. In an industry with price competition, the equilibrium price is expected to exceed the competitive price. Then, increasing the number of firms should lead to a higher price[3]. Janssen and Rasmusen

3 Specifically, the new price is expected to be higher when there is an increase in the size of the market and the number of firms in the same proportion (see Wambach, 1999).

(2002) also consider a Bertrand model with uncertainty on the number of firms operating in the industry. With an uncertain number of competitors, there exists a unique symmetric equilibrium in mixed strategy and again each firm charges a price larger than marginal cost[4]. The question of strategic choices of risk-averse firms is further analyzed in Asplund (2002), who examines how the degree of risk aversion and different types of uncertainty affect competition in an oligopolistic framework. The key feature of this insightful contribution is to propose a general competition model of risk-averse firms that encompasses price competition with differentiated products under various forms of cost and demand uncertainty. In particular, competition is softer in case of marginal cost uncertainty.

Thus, our work may be seen as complementary to the analysis of Asplund (2002). Our contribution is twofold. First, we focus on the consequences of uncertainty in a model with product differentiation and free entry of firms. Second, we present a welfare analysis which accounts for the costs involved by firms in bearing risk. The remainder of the paper is organized as follows. In Sect. 2, we extend the circular location model of Salop (1979) and assume that marginal cost is uncertain. In Sect. 3, we determine the Nash equilibrium in prices for any number of firms and show that firms charge higher prices to consumers because of uncertainty. The Nash equilibrium in the entry game is analyzed in Sect. 4, with a positive impact of uncertainty on the number of firms. Section 5 examines the price equilibrium from a normative viewpoint. Concluding comments are in Sect. 6.

2 The Spatial Model

We consider a model with firms producing differentiated products, in which consumers are heterogeneous and where firms have uncertain marginal costs. Thus, we relax the prevalent assumption behind the Bertrand paradox that firms produce an homogeneous good, a situation analyzed by Janssen and Rasmusen (2002), and Wambach (1999) in an uncertain setting. With a location model, it follows that firms can raise their price above the marginal cost without losing their entire market share.

4 The perfectly competitive equilibrium is the limit case when the number of firms becomes large. As the probability of competition increases, each firm reduces its price.

We restrict our attention to horizontally differentiated products, meaning that brands are not uniformly ranked by all consumers. As usual in the literature, each consumer has a different preference for the brands sold in the market due to different locations. In our setting, location corresponds to the physical location of a particular consumer. Each agent observes the prices charged by all the firms, and then decides to purchase the good from the firm at which the price plus the transportation cost is minimized. Another convenient interpretation is that location can also represent a distance between the brand characteristics viewed as ideal by the consumer and the characteristics of the brand actually purchased[5]. Thus, firms choose their products anticipating that their location decision in product space is expected to affect the intensity of price competition.

Our theoretical analysis of the impact of uncertainty on the number of firms draws on the spatial differentiation model originally described by Salop (1979), corresponding to the case of a circular city. In so doing, we are able to examine the problem of firms' entry on the market given marginal cost uncertainty. Specifically, we study entry and location decisions when there exist no barriers to entry other than fixed costs.

We suppose that consumers are located uniformly on a circle C, which has a perimeter equal to L. Clearly, the circumference L is a measure for the heterogeneity of consumers and it may be seen as an indicator for demand intensity. Individuals are continuously and uniformly distributed along this circumference. We assume without loss of generality that the density is constant, and it is denoted by Δ[6]. Thus, the parameter Δ expresses the thickness of the market. Given the location of firms, consumers incur a transportation cost equal to t per unit of length, such that this cost includes the value of time spent in travel. Each consumer buys exactly one unit of the brand that minimizes the sum of the price and the transportation cost. Nevertheless, this generalized cost has to remain lower than the gross surplus that the consumer can obtain from the good. This outside option is denoted by $\bar{s}$. It is assumed to be large enough, so that the market is always covered in equilibrium (goods are bought by all consumers).

5 In that case, distance is a measure of the disutility from consuming a less-than-ideal product.

6 Relaxing this assumption does not modify our theoretical conclusions. See Calvo-Armengol and Zenou (2002) for the case of a general density in a model of differentiated products, but under certainty.

Firms are located around the circle. Although the circular model of Salop (1979) is a location model, it does not explicitly explain how firms choose their location (see the related discussion in Tirole, 1988, p. 285). Indeed, the spatial model has the following two-stage structure. First, the number of firms is endogenously determined. It is assumed that firms are automatically located at an equal distance from one another. Let n be the number of entering firms, where n is an integer. So, given the circumference L, the distance between any two firms is equal to L/n. Second, firms compete in prices given the previous locations. So, a key feature of this horizontal differentiation model is the focus on firms' entry, and we examine the impact of uncertainty on entry.

To specify the model, we need several assumptions. There are n identical firms, which have all the same technology. To address the issue of entry, we suppose that each firm is characterized by a fixed cost of entry denoted by $\overline{f}$. Once the firm is located at a point on the product space, it faces a marginal cost c that is supposed to be constant. We depart from the model of Salop (1979) by assuming that the marginal cost is uncertain, so that firms face supply-induced cost fluctuations in our setting. To formalize this type of uncertainty, we assume that the marginal cost is described by a random variable $\tilde{c}$ whose mean is $E(\tilde{c}) = c$ and the corresponding variance is $Var(\tilde{c}) = \sigma^2$.[7] As usual, greater cost uncertainty is measured by an increase in the variance σ^2 (a mean preserving spread in costs). We assume that prices are set before marginal costs are realized. This is a standard assumption in the literature introducing risk-averse firms in an oligipoly setting (Asplund, 2002; Wambach, 1999)[8].

We would like to stress that this way to include uncertainty in the location model is not restrictive. Indeed, there are numerous examples in the industry of sources of uncertainty arising by the marginal cost of production. For instance, Wambach (1999, p. 946) mentions the case of insurance corporations where the probability of accident is imperfectly known to the insurers, firms which provide guarantees for new products (given random breakdown), or simply firms which import brands and then face exchange-rate uncertainty. Other explanations concern poor climatic conditions for firms that produce or use agricultural goods or uncertain

7 We assume that there is no correlation between the different firms' marginal costs.

8 Clearly, several simplifying assumptions have to be made to show the effects of risk aversion in an oligopoly framework. See the further discussion in Wambach (1999, pp. 946–947).

wages linked to efficiency wage considerations and shirking behaviors as well as uncertainty about the number of active workers (due to illness).

Each firm is labeled by subscript i ($i = 1, \ldots, n$) and the price set before the state of the world is realized is denoted by p_i. A consumer is located at the distance $x \in C$. Then, the generalized price to buy the brand is equal to $p_i + t|x - x_i|$ under linear transportation costs[9]. Firms anticipate that consumers choose to buy the brands from the firms which give them the lowest full price. In the circular model, a representative firm has only two competitors. Given two levels of prices p_{i-1} and p_{i+1}, the demand pool for the firm i is composed of two sub-segments. The outside boundaries of the pool are given by two marginal consumers, respectively denoted by $\underline{x}$ and $\overline{x}$, for whom the generalized price is identical between two adjacent firms: respectively between $i-1$ and i for $\underline{x}$, and between i and $i+1$ for $\overline{x}$. Thus, the marginal value $\underline{x}$ is the solution of the following equation:

$$p_i + t(x_i - \underline{x}) = p_{i-1} + t(\underline{x} - x_{i-1}). \quad (1)$$

Hence, the consumer which is indifferent between purchasing the brand from firm i and purchasing it from its closest neighbor $i{-}1$ is characterized by:

$$\underline{x} = \frac{(p_i - p_{i-1}) + t(x_i + x_{i-1})}{2t}. \quad (2)$$

So, the firm i faces a demand from all the consumers whose location belongs to the interval $[\underline{x}; x]$, since the generalized price these consumers obtain from firm i is lower than the one they would obtain from firm $i-1$. In a similar way, the marginal consumer $\overline{x}$ is such that $p_i + t(\overline{x} - x_i) = p_{i+1} + t(x_{i+1} - \overline{x})$, which implies:

$$\overline{x} = \frac{(p_{i+1} - p_i) + t(x_i + x_{i+1})}{2t}. \quad (3)$$

Finally, the demand pool for the firm i consists of all consumers whose location is comprised in the closed interval $[\underline{x}; \overline{x}]$.

Now, let Π_i be the profit level of the firm i. Knowing the firm's demand, the presence of a fixed cost and given the uncertainty on

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9 While we restrict our attention to the case of linear transportation costs for the sake of simplicity, our theoretical results remains unchanged with quadratic transportation costs.

marginal cost, the profit for the firm is also a random variable $\tilde{\Pi}_i$ which is given by:

$$\tilde{\Pi}_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}. \tag{4}$$

Given the uncertain environment, we assume that firms are risk averse following some recent extensions in oligopoly theory (see Asplund, 2002; Haruna, 1996; Mai et al., 1993; Tessiotore, 1994; Wambach, 1999). With uncertainty on the marginal cost, we assume that the firm i is characterized by a Von Neumann-Morgenstern utility function denoted by U_i, so that the objective function for the firm may be expressed as

$$\max V_i = E\left[U_i(\tilde{\Pi}_i)\right], \tag{5}$$

where U_i is a continuous, twice-differentiable and concave utility function (meaning that $U_i' > 0$ and $U_i'' < 0$). From the definition of $\tilde{\Pi}_i$, the representative firm i seeks to maximize the expected utility function:

$$V_i = E\left[U_i\left(\Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}\right)\right]. \tag{6}$$

Let us finally remind the definition of the monopolistic-competition equilibrium in the circular city. At the optimum, each firm behaves as a monopoly on its brand, meaning that the firm chooses the price that maximizes its utility function given the demand for brand i and given that all other firms charge the same price, and then free entry of firms results in zero utility. So, we solve the model by first determining the Nash equilibrium in prices for any number of firms, then by calculating the Nash equilibrium in the entry game (see Salop, 1979; Tirole, 1988).

3 The Spatial Differentiation Equilibrium

Let us assume that n firms have entered the market. Since these different firms are located symmetrically around the circle and are identical (i.e., they all have the same cost and utility functions), we examine an equilibrium in which each firm charges the same price. We restrict our attention to the case of a covered market, which means that there are enough firms in the market[10]. In that case, firms are expected to set prices

10 Concerning the issue of covered market in spatial models, see the further discussion of Jellal et al. (1998) in the context of a labor market.

that are not too high. Otherwise, some consumers would no longer purchase anything.

The maximization program for the firm i is $\max_{p_i} V_i$, so that the corresponding first-order condition given by $\partial V_i/\partial p_i = 0$ under marginal cost uncertainty is:

$$E\left[U_i'(.)\left(\Delta(p_i - \tilde{c})\left(\frac{\partial \overline{x}}{\partial p_i} - \frac{\partial \underline{x}}{\partial p_i}\right) + \Delta(\overline{x} - \underline{x})\right)\right] = 0 \tag{7}$$

with $U_i'(.) = U_i'\big(\Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}\big)$ for the notation. We also check that the second-order condition $\partial^2 V_i/\partial p_i^2 < 0$ for a maximum is satisfied since:

$$E\left[U_i''(.)\left(\Delta(p_i - \tilde{c})\left(\frac{\partial \overline{x}}{\partial p_i} - \frac{\partial \underline{x}}{\partial p_i}\right) + \Delta(\overline{x} - \underline{x})\right)^2 + 2\Delta\left(\frac{\partial \overline{x}}{\partial p_i} - \frac{\partial \underline{x}}{\partial p_i}\right)U_i'(.)\right] < 0$$

using $U_i''(.) < 0$ and $\partial \overline{x}/\partial p_i - \partial \underline{x}/\partial p_i < 0$. Since Π_i is continuous in (p_{i-1}, p_i, p_{i+1}) and since Π_i is strictly concave in p_i, we deduce that there always exists a Nash equilibrium in prices and that this Nash equilibrium is unique.

Proposition 1: The symmetric Nash equilibrium price denoted by p_i^* is given by

$$p_i^* = c + \frac{tL}{n} + \frac{cov\left[\tilde{c}, U_i'\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}{E\left[U_i'\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}. \tag{8}$$

Proof: The optimal price is given by condition (7). First, we know that firms are symmetrically located and thus the distance between two firms is L/n, so that the market area for each firm is $\overline{x} - \underline{x} = L/n$. Second, given the definition of the marginal consumers $\overline{x}$ and $\underline{x}$, using (2) and (3) leads to $\partial \overline{x}/\partial p_i - \partial \underline{x}/\partial p_i = -1/t$. Thus, we get:

$$E\left[U_i'(.)\Delta\left(\frac{L}{n} - \frac{p_i - \tilde{c}}{t}\right)\right] = 0.$$

Given the properties of the expectancy operator, it follows that:

$$p_i^* = \frac{tL}{n} + \frac{E\left[\tilde{c}U_i'\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}{E\left[U_i'\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}.$$

Since $\tilde{c}$ is an argument of $U_i'(.)$, we can further simplify the optimal price using the fact that $E(XY) = E(X)E(Y) + cov(X, Y)$ for two variables X and Y. Since the mean of the random marginal cost is $E(\tilde{c}) = c$, we finally deduce (8).

Clearly, the sign of the covariance $cov[\tilde{c}, U_i'(.)]$ is positive, since Baron (1971) has shown that the inequality $cov[\tilde{p}, U_i'(.)] < 0$ holds under price uncertainty and provided that the marginal utility $U_i'(.)$ is decreasing. Proposition 1 gives us a first result concerning the role of cost uncertainty on the spatial differentiation equilibrium. A greater cost uncertainty when producing brands leads to higher generalized prices charged to consumers. In equilibrium, the price p_i^* is the sum of three elements: the marginal cost of production c, the transportation cost tL/n, and the risk premium given by $cov[\tilde{c}, U_i'(.)]/E[U_i'(.)]$.

As the equilibrium price stands, it seems at first sight difficult to interpret the last term dealing with risk aversion. To find a more explicit result and in order to get closed form solutions for our problem (which is necessary to further investigate free entry), we have to make an additional assumption concerning the marginal cost.

Assumption 1: The marginal cost $\tilde{c}$ follows a Normal distribution, with $E(\tilde{c}) = c$ and $Var(\tilde{c}) = \sigma^2$.

The main interest of the normality assumption is that we can use Stein's lemma described in Huang and Litzenberger (1988). Nevertheless, assuming normally distributed marginal costs may appear somewhat problematic since it implies that marginal costs may be negative, even infinitely negative. In fact, we believe that the assumption of normally distributed costs is not so restrictive with respect to the case of nonnegative costs. For instance, the marginal cost may be normally distributed over a closed interval whose lower bound would be strictly positive.[11]

11 Also, some studies have empirically obtained negative marginal costs for some outputs (see Pulley and Humphrey, 1993).

Let us consider two variables X and Y such that they are bivariate normally distributed. If the function $f(Y)$ is continuously differentiable, it has been shown by Rubinstein (1976) that $cov[X, f(Y)] = E[f'(Y)]cov(X, Y)$. Now, if we apply Stein's lemma to our problem, it follows that:

$$cov\left[\tilde{c}, U_i'\left(\Delta(p_i - \tilde{c})L/n - \overline{f}\right)\right] = E\left[U_i''\left(\Delta(p_i - \tilde{c})L/n - \overline{f}\right)\right] \times cov\left[\tilde{c}, \Delta(p_i - \tilde{c})L/n - \overline{f}\right].$$

Since we have $cov[\tilde{c}, \Delta(p_i - \tilde{c})L/n - \overline{f}] = -\Delta\sigma^2 L/n$, this implies:

$$cov\left[\tilde{c}, U'\left(\Delta(p_i - \tilde{c})L/n - \overline{f}\right)\right] = -E\left[U_i''\left(\Delta(p_i - \tilde{c})L/n - \overline{f}\right)\right]\frac{\Delta L}{n}\sigma^2$$

and thus the symmetric Nash equilibrium price may be expressed as[12]

$$p_i^* = c + \frac{tL}{n} - \frac{E\left[U_i''\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}{E\left[U_i'\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}\frac{\Delta L}{n}\sigma^2. \tag{9}$$

Let us define the parameter a_i such that:

$$a_i = -\frac{E\left[U_i''\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}{E\left[U_i'\left(\Delta(p_i^* - \tilde{c})L/n - \overline{f}\right)\right]}.$$

In the literature, the parameter a_i is known as Rubinstein's measure of absolute risk aversion (Rubinstein, 1973; 1976).[13] Lintner (1970) considers the special case of negative exponential utility to show that the measure of risk aversion based on the expectations of $U_i''(.)$ and $U_i'(.)$ is constant and independent of future wealth. In that case, the Rubinstein's measure is simply equal to the Arrow-Pratt measure of absolute risk aversion $-U_i''(.)/U_i'(.)$. For quadratic utility, the Rubinstein measure is

12 The derivation of the first-order condition in the case of normally distributed uncertainty is also obtained in Asplund (2002) as a special case.

13 Asplund (2002, Appendix 1) also uses the measure $-EU_i''(\tilde{\Pi}_i - \overline{f})/EU_i'(\tilde{\Pi}_i - \overline{f})$. The author defines this ratio as the Arrow-Pratt measure of global absolute risk aversion. However, as pointed out by an anonymous referee, this expression cannot be considered as the Arrow-Pratt measure which is given by $-U_i''(\tilde{\Pi}_i - \overline{f})/U_i'(\tilde{\Pi}_i - \overline{f})$.

the inverse of the expected inverse of absolute risk aversion (Mossin, 1969). It is again independent of wealth for constant absolute risk aversion (see Rubinstein, 1974)[14].

Proposition 2: Under assumption 1, the equilibrium price p_i^* is given by:

$$p_i^* = c + \frac{tL}{n} + \frac{\Delta L}{n} a_i \sigma^2 \tag{10}$$

Assumption 1 leads to a closed-form solution for the positive risk premium, which is now equal to $\Delta L a_i \sigma^2 / n$. It is an increasing function of the density Δ of consumers on the circle and of the demand intensity L, but it is negatively related to the number of firms n. In that case, the risk due to uncertain marginal cost is spread over a larger number of firms. Clearly, firms charge higher prices for consumers given cost uncertainty, as shown in Asplund (2002) or Wambach (1999). When firms are characterized by risk aversion ($a_i > 0$), we obtain $\partial p_i^* / \partial a_i = \Delta L \sigma^2 / n > 0$. Also, the price is positively related to the variance σ^2 of the marginal cost since the derivative $\partial p_i^* / \partial \sigma^2 = \Delta L a_i / n$ is positive. Both results indicate that firms share with consumers the risk generated by cost fluctuations. In industries characterized by greater cost uncertainty, higher prices for brands are expected since the risk premium increases.

Another interesting result is that the price p_i^* is an increasing function of the demand intensity L and of the consumer density Δ (only in an uncertain context), with increased opportunities of differentiation for firms. Other findings concerning the variables that affect p_i^* are more standard. With risk-averse firms in the industry ($a_i > 0$), a larger product market exerts a positive effect on the equilibrium price, given the higher possibility of differentiation for firms (the market area for each firm is fixed, given by L/n). Each firm faces the same degree of uncertainty on its marginal cost and the risk premium is an increasing function of the density of consumers, which leads to a higher price. Also, the equilibrium price increases with t since the market power of firms is increased for consumers who are located close to the firms (Salop, 1979). Finally, given the increased competition, we basically observe that the price decreases

14 Properties of this specific measure of risk aversion are further investigated in Li and Ziemba (1993).

with the number of firms in the market since $\partial p_i^*/\partial n = -t/n^2 - \Delta L a_i \sigma^2/n^2 < 0$.[15]

Before finding the equilibrium number of brands (n is endogenous), we briefly examine the situation where firms are risk neutral. When cost fluctuations have no impact on the utility derived by the firms ($a_i = 0$), the equilibrium price is:

$$p_i^* = c + \frac{tL}{n},$$

which is the result obtained by Salop (1979) in a spatial model under certainty.[16] In the restrictive case of risk neutrality, consumer density does not influence the equilibrium price. This conclusion no longer holds true when firms share with consumers part of the risk generated by cost volatility.

4 Free Entry of Firms

We now turn to the determination of the endogenous number of firms n^*, assuming that there are enough potential entrants to cover the market. This implies that uncertainty cannot be too great.[17] By definition, the equilibrium number of firms n^* is given by:

$$E\left[U_i(\tilde{\Pi}_i)\right] = 0. \tag{11}$$

Ignoring Assumption 1, let us suppose more generally that the uncertain cost $\tilde{c}$ is distributed according to a density function $g(\tilde{c})$ defined over the support $\Omega = [\underline{c}; \overline{c}]$ ($\underline{c}$ can be positive). Thus, the previous condition may be expressed as $\int_\Omega U_i[\Pi(\tilde{c})]dg(\tilde{c}) = 0$, the reservation profit being

15 The competitive outcome can be regarded as a limit case of our model, when the number of firms becomes very large.

16 In the original presentation of Salop (1979), the length of the circle is set to one.

17 In the context of our problem, the equilibrium price has to be lower than the gross surplus $\bar{s}$. Since the maximum distance for a consumer is $L/2n$, the corresponding condition of positive surplus is $p^* + \frac{tL}{2n} \leq \bar{s}$, so that the condition ensuring that the market is covered at the price equilibrium is:

$$\sigma^2 < \frac{2n(\bar{s} - c)^2 - 3tL}{2a_i \Delta L}.$$

normalized to 0. Again, the difficulty for our problem is to find an explicit solution for the optimal number of firms n^*, which involves additional restrictions either on the distribution of $\tilde{c}$ or on the functional form for U.

Recall that to derive the equilibrium price p_i^*, we have used Stein's lemma by assuming that the marginal cost was normally distributed. It is well known that the mean and the variance provide a complete characterization of a random variable which is normally distributed. Thus, under assumption 1, we can rely on the mean-variance specification for the utility function U_i.[18] Thus, the problem for a firm is given by:

$$V_i = E(\tilde{\Pi}_i) - \frac{a_i}{2} Var(\tilde{\Pi}_i) - \overline{f}, \qquad (12)$$

where a_i is the degree of absolute risk aversion ($a_i \geq 0$) and the profit is $\tilde{\Pi}_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}$. It follows that:

$$V_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \frac{a_i}{2}(\Delta(\overline{x} - \underline{x}))^2 \sigma^2 - \overline{f}. \qquad (13)$$

One can easily check that with the mean-variance utility, the equilibrium symmetric price is $p_i^* = c + tL/n + \Delta L a_i \sigma^2 / n$ as claimed in Proposition 2. Although firms charge higher prices given cost uncertainty, the problem is that higher uncertainty also comes with a larger risk premium, which is expected to lower the higher profit which can be made. We now prove that the risk premium does not increase as fast as the equilibrium price, which is the source for rising profits with uncertainty.

Starting from (13) which can be expressed as $V_i = \Delta(\overline{x} - \underline{x}) \left(p_i - \tilde{c} - \frac{a_i}{2}\sigma^2 \Delta(\overline{x} - \underline{x})\right) - \overline{f}$, we have to suppose that the following inequality holds:

$$p_i - \tilde{c} > \frac{a_i}{2}\sigma^2 \Delta(\overline{x} - \underline{x}). \qquad (14)$$

Otherwise, the expected utility V_i would be negative. This implies that $\Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) > \frac{a_i}{2}\sigma^2 \Delta^2 (\overline{x} - \underline{x})^2$. We know that $\Delta(p_i - \tilde{c})(\overline{x} - \underline{x})$ is equal to $E(\Pi)$ and that $\frac{a_i}{2}\sigma^2 \Delta^2 (\overline{x} - \underline{x})^2$ is the risk premium supported by the firm. Hence, from inequality (14), the mean margin per unit has to be

18 The mean-variance approach can be used if the stochastic distribution of the marginal cost belongs to a particular parametrized family, normal or elliptical random variable. On the relevance of this approach, see Liu (2003).

greater than the mean risk premium. Let us further examine the effect of p_i on V_i. From the first-order condition $\partial V_i / p_i = 0$, we obtain:

$$p_i^* = c + \frac{a_i}{2}\sigma^2 \frac{\Delta L}{n} + \frac{tL}{n}\left(1 + \frac{a_i}{2}\sigma^2 \frac{\Delta}{t}\right).$$

Hence, the first and second terms are the costs supported by the firms, where c is the marginal cost and $\frac{a_i}{2}\sigma^2 \frac{\Delta L}{n}$ is the mean risk premium. The third term which is equal to $\frac{tL}{n} + \frac{a_i}{2}\sigma^2 \frac{\Delta L}{n}$ is the rent per unit, and it may be decomposed as follows. On the one hand, $\frac{tL}{n}$ is the rent owing to spatial differentiation. On the other hand, $\frac{a_i}{2}\sigma^2 \frac{\Delta L}{n}$ is the mean risk premium. Hence, we find that the equilibrium price comprises twice the risk premium. This result occurs because of the form of the utility function, firms being risk averse. This point is further investigated in the appendix, where we consider a more general case and prove that the effect of uncertainty depends on risk aversion. In our mean-variance case, we find that the mean premium supported by the firm is $\frac{a_i}{2}\sigma^2 \frac{\Delta L}{n}$, while the marginal premium charged to consumers is twice as high and equal to $a_i \sigma^2 \frac{\Delta L}{n}$.

This clearly explains why the risk premium does not increase as fast as the equilibrium price. In fact, the difference between the cost of higher uncertainty supported by firms and the cost charged to consumers depends on the magnitude of risk aversion.[19] If uncertainty increases, the risk premium of the marginal buyer who determines the equilibrium price increases by more than the risk premium of the average buyer, which determines the risk premium that the firms requires to bear the risk. Thus, with more uncertainty, the price increases by more than the risk premium and firms are strongly induced to enter the market.

Let us formally prove that result. Using the optimal value for p_i^*, we finally obtain V_i such that:

$$V_i = t\Delta\left(\frac{L}{n}\right)^2 + \frac{a_i}{2}\sigma^2\Delta^2\left(\frac{L}{n}\right)^2 - \overline{f}. \tag{15}$$

19 A similar problem may be found in the circular model without uncertainty, as one considers a general cost function of the type $C = C(\Delta(\overline{x} - \underline{x}))$. In that case, entry of firms depends on the comparison between the marginal cost $C'(q)$ and the mean cost $C(q)/q$.

Since the number of firms n^* is given by $V_i(n^*) = 0$, we get $\left(\frac{L}{n}\right)^2 \left(t\Delta + \frac{a_i}{2}\Delta^2\sigma^2\right) = \overline{f}$. As it is defined, the equilibrium number of firms is not necessarily an integer.

Proposition 3: Under Assumption 1 and with a mean-variance utility function, the number of firms n^* in a situation of imperfect competition with free entry is:

$$n^* = \sqrt{\frac{(t\Delta + \frac{a_i}{2}\Delta^2\sigma^2)L^2}{\overline{f}}}. \tag{16}$$

Also, $n^* > n_0^*$ holds, $n_0^* = \sqrt{t\Delta L^2/\overline{f}}$ being the number of firms under certainty.

Proposition 4: Under Assumption 1 and with a mean-variance utility function, the price p^* under free entry is given by

$$p^* = c + \sqrt{\frac{t\overline{f}}{\Delta}}\sqrt{\frac{(1 + a_i\sigma^2\frac{\Delta}{t})^2}{(1 + \frac{a_i}{2}\sigma^2\frac{\Delta}{t})}}. \tag{17}$$

Clearly, we have $(1 + a_i\sigma^2\frac{\Delta}{t})/\sqrt{(1 + \frac{a_i}{2}\sigma^2\frac{\Delta}{t})} > 1$. Since the equilibrium price under certainty p_0^* is simply $p_0^* = c + \left(\frac{t\overline{f}}{\Delta}\right)^{\frac{1}{2}}$, we can easily compare p_0^* and p^*.

Corollary 1: With free entry of firms, the price is higher under uncertainty.

In this model of spatial differentiation, the main contribution of our paper is to formally prove that greater uncertainty increases the number of firms in an industry. There are more firms because of uncertainty *and* risk aversion.[20] Clearly, both the degree of risk aversion a_i and the measure of variance σ^2 exert a positive effect on the optimal number of firms. That uncertainty positively affects free entry may be surprising, since it is usually admitted that greater uncertainty is rather expected to decrease the number of firms in an industry. For instance,

20 If the degree of risk aversion a_i is set to 0 (or $\sigma^2 = 0$), we find that the number of firms is $n^* = \sqrt{t\Delta L^2/\overline{f}}$, which is the original result of Salop (1979).

in the context of price uncertainty, Sandmo (1971) argues that firms characterized by a large risk aversion will choose not to enter in an industry facing a high degree of uncertainty. Only low risk-averse firms are expected to enter in industries with greater uncertainty, thereby reducing the number of firms.

Then, how can we justify that greater uncertainty does not act as a barrier to entry under spatial competition? In fact, we have previously shown that firms can charge a higher price to consumers under marginal cost uncertainty. They shift the risk to the consumers and although the risk premium becomes larger with greater uncertainty, the price includes twice the risk premium in the mean-variance case (see the Appendix). Risk-averse firms have therefore greater incentives to enter the market. This positive relationship between entry and uncertainty under spatial differentiation is a novel result with respect to the previous literature for models in which the number of firms in the market is endogenously determined.[21]

With respect to related studies, we would like to stress that our setting is different since we rely on a model of spatial differentiation instead of assuming the presence of homogeneous goods. Nevertheless, the increased number of firms under free entry is not really due to spatial differentiation. In fact, we prove in this paper that more uncertainty leads to more firms exactly in those cases where the marginal buyer requires a larger risk premium than the average buyer. Otherwise, more uncertainty would reduce the number of firms in the industry.

5 Welfare Analysis

We now consider the price equilibrium under uncertainty from a normative viewpoint. In particular, we examine the impact of marginal cost uncertainty in a free-entry and exit equilibrium in order to know whether uncertainty produces a larger or a smaller variety of brands than the optimal variety level.[22]

21 Also, we observe that an increase in the fixed cost causes a decrease in the number of firms in the market and that a rise in the transportation cost leads to an increase in the profit margin since there is a higher probability of differentiation for firms.

22 Under certainty, it is well known that private and social incentives do not necessarily coincide and the market is expected to generate too many firms (see Tirole, 1988).

With respect to the previous discussion, we have to account for the additional cost involved in bearing risk since firms are risk-averse. From the definition of V_i such that $V_i = E(\tilde{\Pi}_i) - \frac{a_i}{2} Var(\tilde{\Pi}_i) - \overline{f}$, we note that the term $\frac{a_i}{2} Var(\tilde{\Pi}_i)$ indicates the risk supported by each firm given the randomness of $\tilde{\Pi}_i$. Using the definition of the profit level $\tilde{\Pi}_i$, we deduce that $Var(\tilde{\Pi}_i) = \Delta^2 L^2 \sigma^2 / n^2$. Thus, the cost of risk bearing by a firm denoted by B_i is given by

$$B_i = \frac{a_i}{2} \left(\frac{\Delta L}{n} \right)^2 \sigma^2. \tag{18}$$

We note that this cost increases with the absolute degree of risk aversion a, with the demand intensity L and with the variance of the marginal cost σ^2. Conversely, risk bearing costs are a decreasing function of the number of firms n. The aggregate cost of risk bearing is simply nB_i.

In the spatial model of Salop (1979), the aggregate transportation cost T is:

$$T = 2nt \int_0^{L/2n} \Delta x dx, \tag{19}$$

since all consumers purchasing the brand from a firm are located between 0 and $L/2n$ units of distance from that firm. So, the average consumer has to travel $L/4n$ units of distance, which leads to the following aggregate transportation cost:

$$T = \frac{t\Delta L^2}{4n}. \tag{20}$$

Now, the problem for the social planner is to minimize the sum of fixed costs paid by the producing firms, aggregate transportation costs and aggregate costs of risk bearing. The social aggregate cost S is then equal to $S = n\overline{f} + T + nB_i$. Formally, the problem for the social planner may be expressed as

$$\min_n n\overline{f} + \frac{t\Delta L^2}{4n} + \frac{a_i}{2} \frac{(\Delta L)^2}{n} \sigma^2. \tag{21}$$

Proposition 5: Under cost uncertainty, the optimal number of firms $\hat{n}$ chosen by an omniscient planner is

$$\hat{n} = \sqrt{\frac{L^2}{\bar{f}}\left(\frac{t\Delta}{4} + \frac{a_i}{2}\sigma^2\Delta^2\right)}. \tag{22}$$

Proof: Since the problem for the social planner is $\min_n S$, we solve the corresponding first-order condition $\partial S/\partial n = 0$ and obtain:

$$\bar{f} - \frac{1}{\hat{n}^2}\left(\frac{t\Delta L^2}{4} + \frac{a_i}{2}\sigma^2(\Delta L)^2\right) = 0,$$

which gives the optimal number of firms $\hat{n}$. □

Corollary 2: The market generates too many firms at the equilibrium, i.e., $\hat{n} < n^*$.

When comparing the number of firms chosen by the social planner and the decentralized equilibrium, it follows that:

$$\hat{n} < n^* = \sqrt{\frac{L^2}{\bar{f}}\left(t\Delta + \frac{a_i}{2}\sigma^2\Delta^2\right)}. \tag{23}$$

In the free-entry location model, the market generates too many firms in equilibrium. Clearly, too many brands are produced since firms have too much of an incentive to enter. Of course, such a result also holds in the model of Salop (1979) under certainty. As compared to spatial differentiation under certainty, we observe that the social planner chooses a higher number of firms in order to achieve an optimal risk-sharing among firms. Increasing the number of firms in the market leads to an implicit hedging. Finally, when the transportation cost is very low, we find that n^* is approximately equal to $\hat{n}$. In that case, the number of firms only depends on costs involved in bearing risk, and this factor, which is equal to $\frac{a_i}{2}\sigma^2\Delta^2$, is identical in n^* and $\hat{n}$.[23]

23 When $t \to 0$, we get $n^* = \hat{n} = \sqrt{\frac{a_i}{2}\frac{\sigma^2\Delta^2 L^2}{\bar{f}}}$.

To highlight the impact of n on the social aggregate cost, let us comment the following derivative $\partial S(n)/\partial n = \overline{f} - \frac{1}{n^2}\left(\frac{t\Delta L^2}{4} + \frac{a_i}{2}\sigma^2(\Delta L)^2\right)$. The first term is positive and is due to fixed costs. Conversely, the second term is negative; it indicates that costs will be reduced for consumers since there is now a greater variety of products. In the same vein, the third term is negative and it may be seen as the reduction of the burden involved by cost uncertainty. Hence, regulating the number of firms may be seen as a hedging device for the social planner. So, there are two positive effects: consumers will pay lower prices, and costs involved in bearing risk will be lower for firms. Clearly, an extension of this analysis should account for hedging markets.

Since entry of firms is socially justified by the savings in transportation costs and costs of risk bearing, we suggest that there are some policy solutions for the social planner in order to reduce the excessive entry of firms in the market. In particular, any policy designed to decrease the level of risk in industries may be an effective way to regulate the market. Resources devoted to the pooling of industrial risks should significantly contribute to the decline of prices charged by firms, by lessening the production risk premium supported by consumers when buying the goods given spatial differentiation.

6 Conclusions

In this paper, we have analyzed a location model to examine the effects of uncertainty in an industry equilibrium. We extend the model of spatial differentiation described in Salop (1979) by introducing marginal cost uncertainty, and we examine the free-entry equilibrium. Accounting for horizontal product differentiation strongly affects the effects of uncertainty on the number of firms in an industry, which is indeterminate in a standard framework with homogeneous goods and price uncertainty (Appelbaum and Katz, 1986). Our analysis is a contribution to the recent developments on the theory of oligopolistic firms under uncertainty with differentiated products, presented in Asplund (2002).

In our setting, the optimal price charged to consumers includes an additional term owing to uncertainty. With the mean-variance case, we find that the price charged to consumers includes twice the mean risk premium supported by the firms, so that the cost of uncertainty is supported by consumers with differentiation. Since the risk premium of the marginal buyer is larger than the risk premium of the average buyer, the price increases by more than the risk premium with more uncertainty and incentives to enter the market are increased. Finally, comparing the number of goods in a market economy and a social economy indicates that too many brands are produced in a free-entry location model, cost uncertainty having an additional positive impact on the distortion.

A final comment deals with empirical testing. Our framework suggests a positive relationship between cost uncertainty and entry of firms in industries with differentiated products. However, evidence on the effects of uncertainty on the industry equilibrium remains scarce. Using a cross-section of American manufacturing industries, Ghosal (1996) finds that greater price uncertainty has a significant and large negative effect on the number of firms in an industry. Focusing on the intertemporal dynamics of industry structure again for manufacturing firms in the United States, Ghosal (2002) shows that greater uncertainty does not affect large establishments, while it has a negative impact on the number of small firms in an industry (see also Ghosal and Loungani, 2000).

Nevertheless, this observed negative relationship between uncertainty and industry equilibrium should not necessarily be interpreted against our model of spatial competition. For instance, Ghosal (1996) only includes a price uncertainty measure and does not account for cost uncertainty. Asplund (2002) clearly shows that different types of uncertainty may have opposite effects on competition for risk-averse firms in oligopolies. Also, the issue of differentiated products is not specifically addressed in the previous empirical literature. Thus, it would be useful to investigate the effects of uncertainty on the number of firms for markets with differentiated products and significant cost uncertainty. Such markets could be identified with uncertainty measures based on the standard deviations of residuals in price equations for most important inputs. This empirical issue, which could provide valuable information for public policy, is left for future research.

Appendix

In this appendix, we formally prove that more uncertainty leads to a higher number of firms only if the risk premium of the marginal buyer is larger than the risk premium of the average buyer. For that purpose, we consider the general utility function

$$U_i = E(\tilde{\Pi}_i) - \frac{a_i}{2}\left[\sigma(\tilde{\Pi}_i)\right]^r,$$

where r is a parameter measuring relative risk aversion. With respect to our previous formal analysis, we can see that the case $r = 2$ is the mean-variance utility function. The cost of uncertainty supported by a firm is

$$\psi_r = \frac{a_i}{2}\left(\sigma \frac{\Delta L}{n}\right)^r.$$

The problem for the firm is given by

$$V_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \frac{a_i}{2}(\sigma\Delta(\overline{x} - \underline{x}))^r - \overline{f}.$$

After some manipulations, we deduce that the value for the equilibrium price is:

$$p_i^* = c + \frac{tL}{n} + r\frac{a_i}{2}\sigma^r\left(\frac{\Delta L}{n}\right)^{r-1}$$

which can also be expressed as

$$p_i^* = c + \frac{tL}{n} + r\frac{a_i}{2}\left(\frac{\sigma\Delta L}{n}\right)^r\left(\frac{\Delta L}{n}\right)^{-1}.$$

The mean risk premium is given by $\psi_r/(\Delta L/n)$. The definition of the equilibrium price implies that the firm charges r times the mean risk premium to the consumers. Hence, depending on the value of the parameter r, we find that firms are in a position to charge more risk to consumers than they have to support.

At the equilibrium, the level of expected utility conditional on the parameter r is

$$V^r(n) = t\Delta\left(\frac{L}{n}\right)^2 + (r-1)\frac{a_i}{2}\left(\sigma\Delta\frac{L}{n}\right)^r - \overline{f},$$

so that the equilibrium number of firms n^* is given by

$$t\Delta\left(\frac{L}{n^*}\right)^2 - \overline{f} + (r-1)\frac{a_i}{2}\left(\sigma\Delta\frac{L}{n^*}\right)^r = 0.$$

It is then straightforward to interpret the general case. The number of firms in the free entry game depends on the value of the parameter r. As shown in Wagener (2004), r may be seen as a measure of risk attitudes. Decreasing, constant and increasing relative risk aversion are respectively reflected as $r < 1$, $r = 1$ and $r > 1$. When $r = 1$, we find that $n_{r=1} = \left(t\Delta L^2/\overline{f}\right)^{\frac{1}{2}}$, which is the original case of Salop (1979). With $r = 2$, which is the mean-variance case further examined in our paper, we obtain $n^* = \left(\frac{(t\Delta + \frac{a_i}{2}\Delta^2\sigma^2)L^2}{\overline{f}}\right)^{\frac{1}{2}}$ and firms charge to the consumers twice the mean premium. Clearly, this general formulation indicates that firms have greater incentives to enter the market only when the inequality $r > 1$ holds. In that case, the risk premium of the marginal buyer increases by more than the risk premium of the average buyer.

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